Decentralized supervisory control of nondeterministic discrete event systems: The existence condition of a robust and nonblocking supervisor

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Abstract

This paper addresses a decentralized supervisory control problem for an uncertain discrete event system (DES) modeled by a set of possible nondeterministic automata with unidentified internal events. For a given language specification, we present the existence condition of a robust and nonblocking decentralized supervisor that achieves this specification for any nondeterministic model in the set. In particular, we show that the given language specification can be achieved based on the properties of its controllability and coobservability with respect to the overall nominal behavior of the uncertain DES. It is further shown that the existence of a nonblocking decentralized supervisor can be examined with a trajectory model of the language specification.

Keywords: Decentralized supervisory control; Nondeterministic discrete event systems; Robust supervisors; Nonblockingness; Trajectory models

1. Introduction

In decentralized supervisory control of a discrete event system (DES), there have been studies on various decision structures and the properties of a given language specification in order to design a set of local supervisors achieving the given specification as in Rudie and Wonham (1992) and Yoo and Lafortune (2002). In particular, Yoo and Lafortune (2002) have presented a general decentralized supervisory control structure based on conjunctive and disjunctive fusion rules for local decisions. Most of the previous results on decentralized supervisory control assumed however that there is no model uncertainty and all state transitions are deterministic for a DES to be controlled. Park and Lim (2000) have further investigated the decentralized supervisory control problem under model uncertainty; however, it has been done only for an uncertain DES modeled by a set of possible deterministic automata upon the conjunctive fusion rule.

In this paper, we explore the existence condition of a decentralized supervisor that can achieve a given language specification for an uncertain DES modeled by a set of possible nondeterministic automata with unidentified internal events. The present study is based on the previous work of Park and Lim (2002) in which the existence condition of a robust and nonblocking decentralized supervisor can be examined with a trajectory model of the language specification.

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DESs based on various strategies (e.g., a lifting algorithm that transforms a nondeterministic automaton into an expanded deterministic one). Most of the previous results have commonly shown that the trajectory modeling formalism is an effective way of completely describing the behavior of nondeterministic DESs. The trajectory models are therefore employed in this paper and, based on the trajectory model of a given language specification, we present the existence condition of a robust and nonblocking decentralized supervisor that can achieve the language specification for any nondeterministic model in the set of uncertain models.

2. Preliminaries

We consider an uncertain automaton modeled by a set of all possible nondeterministic automata as follows: \( G := \{ G_1, \ldots, G_k \} \) with \( G_l = (\Sigma_l, Q_l, \delta_l, q_1^l, Q_f^l) \) where \( \Sigma \) is the set of events, \( Q_l \) is the set of states, \( \delta_l: Q_l \times \Sigma \rightarrow 2^Q_l \) (a power set of \( Q_l \)) is the nondeterministic transition function, \( q_1^l \) is the initial state, and \( Q_f^l \subseteq Q_l \) is the set of marked states. The events set \( \Sigma \) is further decomposed into \( \Sigma = \Sigma_n \cup \{ A \} \) in which \( \Sigma_n \) is the nominal identified events set and \( A \) represents an unidentified internal event. In general, \( e \) is often used to represent an internal event in nondeterministic DESs. However, \( e \) also represents a null string, and therefore in order to avoid any confusion, we use \( A \) in this paper to denote an internal event instead of \( e \). We assume that a system cannot undergo an unbounded number of internal transitions. Let \( \Sigma^* \) denote the set of all finite strings over \( \Sigma \) including \( e \). For a language \( L \subseteq \Sigma^* \), \( pr(L) := \{ u \in \Sigma^* \mid uv \in L \text{ for some } v \in \Sigma^* \} \), and \( L \) is called prefix-closed if \( L = pr(L) \). The closed behavior and the marked behavior of \( G_l \) are defined as \( L(G_l) := \{ s \in \Sigma^* \mid \delta_l(q_1^l, s) \neq \emptyset \} \) and \( m_l(G_l) := \{ s \in \Sigma^* \mid \delta_l(q_1^l, s) \cap Q_f^l \neq \emptyset \} \), respectively. In addition, we define a projection \( P_n: \Sigma^* \rightarrow \Sigma_n^* \) which erases the event \( A \) from a string in \( \Sigma^* \).

The trajectory model is known as being able to characterize the behavior of nondeterministic DESs in full detail (Kumar & Shyamkumar, 1996). A trajectory is an element of \( 2^{2\Sigma} \times (\Sigma \times 2^{2\Sigma})^* \) in the form of \( t = (X_0, (\sigma_1, X_1), \ldots, (\sigma_n, X_n)) \) where \( \sigma_l \) denotes the ith executed event and \( X_i \) denotes the set of events refused after the ith executed event, i.e., \( X_i := \{ x \in \Sigma_n \mid \delta_l(q_1^l, q_1^l, \ldots, q_1^l, x) = \emptyset \} \) (\( \sigma_0 = e \)). In this paper, we assume that the ith executed event \( \sigma_l \) can be an internal event \( A \), but the ith refused events set \( X_i \) only contains the nominal events. For a trajectory \( t \), \( \text{tr}(t) := \sigma_1 \cdots \sigma_n \in \Sigma^* \) is called a trace of \( t \). The prefix closure of a trajectory set \( T \subseteq 2^{2\Sigma} \times (\Sigma \times 2^{2\Sigma})^* \) is defined as \( pr(T) := \{ t \in 2^{2\Sigma} \times (\Sigma \times 2^{2\Sigma})^* \mid t' \in T \text{ for some } t' \in (\Sigma \times 2^{2\Sigma})^* \} \).

For \( q \in Q_l \), let \( X(q) := \{ x \in \Sigma_n \mid \delta_l(q, x) = \emptyset \} \). Now, we define a trajectory model \( T(M, G_l) \) of a language \( M \subseteq \Sigma^* \) with respect to \( (w.r.t.) G_l \in G \) as follows:

(i) \( X(q_1^l) \in T(M, G_l) \);
(ii) \( \text{Let } t \in T(M, G_l) \text{ and } \text{tr}(t) \in pr(M) \text{ for } t \in 2^{2\Sigma} \times (\Sigma \times 2^{2\Sigma})^* \).

Then, for \( \sigma \in \Sigma \) and \( \Sigma' \subseteq \Sigma_n, t(\sigma, \Sigma') \in T(M, G_l) \) if there exists a \( q \in \delta_l(q_1^l, \text{tr}(t)\sigma) \) such that \( \text{tr}(t)\sigma \in pr(M) \) and \( X(q) = \Sigma' \).

This implies that the trajectory model \( T(M, G_l) \) is the set of trajectories each of which consists of a trace belonging to \( pr(M) \) and refused event sets defined at states on a path of the trace in \( G_l \). It follows from the definition that \( T(M, G_l) = pr(T(M, G_l)) \).

We consider in this paper a robust supervisory control problem based on the general decentralized supervisory control structure of Yoo and Laforlume (2002). In the decision fusion scheme, the trajectory model is defined as follows:

\[
T_m(G_l) := \{(X(q^l_1), (\sigma_1, X(q^l_1)), \ldots, (\sigma_j, X(q^l_j))) \mid 2^{\Sigma^*} \times (\Sigma \times 2^{\Sigma^*})^* | (q^l_0, \sigma_1, \ldots, q^l_j) \}\] is a marked path of \( G_l \).

We assume that for any two different paths of \( G_l \), if the corresponding trajectories are identical and one of the paths is marked, then the other path must be marked as well.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig.1.png}
\caption{The decentralized supervisory control structure for the uncertain DES.}
\end{figure}
cal decisions over $\Sigma_{c,d}$ are processed by the disjunctive fusion rule. In addition, we let $\Sigma_{c,d,i} := \Sigma_{c,d} \cap \Sigma_{c,i}$ and $\Sigma_{c,e,i} := \Sigma_{c,e} \cap \Sigma_{c,i}$.

For $K \subseteq \Sigma_{c,e}^*$ and $s \in pr(K)$, let the estimation set $E_i(s) := P_i^{-1} P_i(s) \cap pr(K)$. In order to achieve a given language specification $K \subseteq \bigcup_{i=1}^{k} P_n[L_m(G_i)]$, each local supervisor $S_i$ can be designed as follows: for $t \in \bigcup_{i=1}^{k} L(G_i)$,

$$S_i(P_i(t)) := \left\{ \sigma \in \Sigma_{c,e,i} \mid E_i(t) \sigma \cap \bigcap_{i=1}^{k} P_n[L(G_i)] \right\} \subseteq pr(K)$$

$$\cup \{ \sigma \in \Sigma_{c,e,i} \mid E_i(t) \sigma \cap pr(K) \neq \emptyset \}$$

$$\cup (\Sigma_{c,e} \cup \Sigma_{c,e,i}) \cup \Sigma_{uc}.$$  

This implies that the decision for the events in $\Sigma_{c,e,i}$ follows the permissive rule and the decision for the events in $\Sigma_{c,d,i}$ follows the anti-permissive rule w.r.t. the union of nominal behaviors of possible non-deterministic models. Based on the local decisions, the decentralized supervisor $S_{dec}$ is defined by

$$S_{dec}(t) := P_{\Sigma_{c,e}} \left[ \bigwedge_{i} S_i(P_i(t)) \right] \cup P_{\Sigma_{c,d}} \left[ \bigvee_{i} S_i(P_i(t)) \right] \cup \Sigma_{uc},$$

where $P_{\Sigma_{c,e}}$ and $P_{\Sigma_{c,d}}$ are projection mappings: $P_{\Sigma_{c,e}} : \Sigma^* \mapsto \Sigma_{c,e}^*$ and $P_{\Sigma_{c,d}} : \Sigma^* \mapsto \Sigma_{c,d}^*$. That is, the fusion of local decisions on the events in $\Sigma_{c,e}$ follows the conjunctive rule and that on the events in $\Sigma_{c,d}$ follows the disjunctive rule. The behavior of the supervised system $S_{dec}/G_i$ is then defined recursively as follows:

$$[t \in L(S_{dec}/G_i)] \land [\sigma \in L(G_i)] \land [\sigma \in S_{dec}(t)]$$

$$\Leftrightarrow \exists \tau \in L(S_{dec}/G_i).$$

We briefly summarize the notions of controllability and coobservability of a given language (Yoo & LaFortune, 2002). Let $L_c(\sigma) := \{ i \in I_S \mid \sigma \in \Sigma_{c,i} \}$. Then, for a given partition $\Sigma_c = \Sigma_c \cup \Sigma_{c,d}$, $K(\subseteq M(=pr(M)) \subset \Sigma_c^*)$ is said to be coobservable w.r.t. $M$ and $(\Sigma_{c,i}, \Sigma_{c,d,i}, \Sigma_{c,e,i})$ if (i) $D \& A$ (disjunctive and anti-permissive) coobservable: $\forall s \in pr(K)$ and $\forall \sigma \in \Sigma_{c,d}$ with $s \sigma \in pr(K)$, there exists $i \in L_c(\sigma)$ such that (s.t.) $E_i(s) \sigma \cap M \subseteq pr(K)$, and (ii) $C \& P$ (conjunctive and permissive) coobservable: $\forall s \in pr(K)$ and $\forall \sigma \in \Sigma_{c,e}$ with $s \sigma \in M \cap pr(K)$, there exists $i \in L_c(s) \cap M \cap pr(K) \neq \emptyset$. In addition, $K$ is said to be controllable w.r.t. $M$ if $pr(K) \Sigma_{uc} \cap M \subseteq pr(K)$.

3. Main results

For some non-deterministic model $G_i \in G$, even though the supervised system $S_{dec}/G_i$ is language-model nonblocking (i.e., $L(S_{dec}/G_i) = pr(L(S_{dec}/G_i) \cap L_m(G_i)))$, it can actually get blocked (Kumar & Shyamam, 1996). Hence, we need to redefine the nonblockingness of a supervisor for non-deterministic DESs. While Kumar and Shyamam (1996) and Heymann and Lin (1996) have proposed the notion of trajectory-model nonblockingness in the absence of uncertainty, we further introduce the following definition for the uncertain DESs in order to get more intuitive understanding:

**Definition 1.** A decentralized supervisor $S_{dec}$ is said to be robust and nonblocking for an uncertain nondeterministic DES $G := \{G_1, \ldots, G_k\}$ if, for all $G_i \in G$ and for all $s \in L(S_{dec}/G_i)$ and $q \in \delta_i(q_0, s)$, there exists $s' \in \Sigma^*$ such that $ss' \in L(S_{dec}/G_i)$ and $\delta_i(q, s') \cap Q_i^m \neq \emptyset$.

The above definition implies that a decentralized supervisor $S_{dec}$ is robust and nonblocking for the uncertain DES $G$ represented by a set of possible nondeterministic models if there exist strings leading to some marked states from all possible states of the supervised systems for all of the models. Based on this notion, we address the following problem in this paper:

- Given a nonempty $K \subseteq \bigcup_{i=1}^{k} P_n[L_m(G_i)]$, find the existence condition of a robust and nonblocking decentralized supervisor $S_{dec}$ for the uncertain nondeterministic DES $G := \{G_1, \ldots, G_k\}$ such that $P_n[L(S_{dec}/G_i)] = pr(K)$ for all $G_i \in G$.

The robust and nonblocking supervisor presented in this paper is an extension of the nonblocking supervisor proposed by Ramadge and Wonham (1987) to a more general class of uncertain and nondeterministic systems. Thus, if a decentralized supervisor $S_{dec}$ is robust and nonblocking for an uncertain nondeterministic DES $G := \{G_1, \ldots, G_k\}$, then it follows that $L(S_{dec}/G_i) = pr(L(S_{dec}/G_i) \cap L_m(G_i))$ for all $G_i \in G$, i.e., $S_{dec}$ is nonblocking for all $G_i \in G$ in the sense of Ramadge and Wonham (1987). This is because $s \in pr(L(S_{dec}/G_i) \cap L_m(G_i))$ in Definition 1 since, for any $s \in L(S_{dec}/G_i)$, there exists $s' \in \Sigma^*$ such that $ss' \in L(S_{dec}/G_i)$ and $\delta_i(q, s') \cap Q_i^m \neq \emptyset$, which implies $L(S_{dec}/G_i) \subseteq pr(L(S_{dec}/G_i) \cap L_m(G_i))$. Since $L(S_{dec}/G_i) \supseteq L(S_{dec}/G_i) \cap L_m(G_i)$, the equality of the two languages holds. However, the fact that $S_{dec}$ is nonblocking for all $G_i \in G$ in the sense of Ramadge and Wonham does not necessarily imply the robust nonblockingness of the supervisor. Hence, the proposed concept of robust and nonblocking supervisors can be regarded as a generalization of the Ramadge and Wonham’s concept without weakening the original notion of markings. In addition, the proposed notion of a robust and nonblocking supervisor is more useful since it also integrates the requirement of achieving a given language specification $K$, i.e., $P_n[L(S_{dec}/G_i)] = pr(K)$ for all $G_i \in G$, within one framework.

Before we further investigate the problem formulated in the above, we introduce the following lemma.

**Lemma 1.** For any $s \in P_n^{-1}(pr(K))$ and $q \in \delta_i(q_0, s)$ in $G_i \in G$, there exists $s' \in \Sigma^*$ such that $ss' \in P_n^{-1}(pr(K))$ and $\delta_i(q, s') \cap Q_i^m \neq \emptyset$

$$\Leftrightarrow T(P_n^{-1}(pr(K)), G_i) = pr(T(P_n^{-1}(pr(K)), G_i) \cap T_m(G_i)).$$
Proof. (If) We can prove it by showing a contradiction as follows:

For some \( s \in P_{n}^{-1}(pr(K)) \) and \( q \in \delta_{1}(q_{0}^{0}, s) \), there is no \( s' \in \Sigma^{*} \) such that \( ss' \in P_{n}^{-1}(pr(K)) \) and \( \delta_{1}(q, s') \cap Q_{n}^{m} \neq \emptyset \)

\[ \Rightarrow \delta_{1}(q, s') \cap Q_{n}^{m} = \emptyset \] for any \( s'' \in \Sigma^{*} \) such that

\[ ss'' \in P_{n}^{-1}(pr(K)) \]

There exists a path \( p = (q_{1}^{0}, \sigma_{1}, q_{1}, \sigma_{2}, q_{2}, \ldots, \sigma_{j}, q) \) of \( G_{1} \) such that:

(i) \( \sigma_{1} \cdots \sigma_{j} = s; \)

(ii) \( t = (X(q_{j}^{0}), \sigma_{1}, X(q_{1})), \ldots, (\sigma_{j}, X(q)) \) \( \in T(p^{-1}(pr(K)), G_{1}); \)

(iii) \( t' \notin T_{m}(G_{1}) \) for any trajectory \( t' \) such that \( t' \in T(p^{-1}(pr(K)), G_{1}) \)

\[ \Rightarrow t \in T(P_{n}^{-1}(pr(K)), G_{1}) \backslash pr(T(P_{n}^{-1}(pr(K)), G_{1}) \cap T_{m}(G_{1})); \]

Contradiction.

(Only if) Since \( T(P_{n}^{-1}(pr(K)), G_{1}) \) is prefix-closed, it suffices to show that \( T(P_{n}^{-1}(pr(K)), G_{1}) \subseteq pr(T(P_{n}^{-1}(pr(K)), G_{1}) \cap T_{m}(G_{1})). \) For \( t \in \Sigma^{2n} \times (\Sigma \times 2^{n}), \)

\[ t \in T(P_{n}^{-1}(pr(K)), G_{1}) \]

There exists a path \( p = (q_{1}^{0}, \sigma_{1}, q_{1}, \sigma_{2}, q_{2}, \ldots, \sigma_{j}, q) \) of \( G_{1} \) such that

\[ t = (X(q_{j}^{0}), \sigma_{1}, X(q_{1})), \ldots, (\sigma_{j}, X(q)) \] 

(by the definition of \( T(P_{n}^{-1}(pr(K)), G_{1}) \))

\[ \Rightarrow \] If we let \( \sigma_{1} \cdots \sigma_{j} = s, \)

\( q \in \delta_{1}(q_{0}^{0}, s) \) and there exists \( s' \in \Sigma^{*} \) such that

\[ ss' \in P_{n}^{-1}(pr(K)) \) and \( \delta_{1}(q, s') \cap Q_{n}^{m} \neq \emptyset \)

There exists a path \( p = (q_{1}^{0}, \sigma_{1}, q_{1}, \sigma_{2}, q_{2}, \ldots, \sigma_{j}, q, \sigma_{j+1}, \ldots, \sigma_{m}, q_{m}) \) of \( G_{1} \) such that \( \sigma_{j+1} \cdots \sigma_{m} = s' \) and \( q_{m} \in Q_{n}^{m} \)

\[ t = ((\sigma_{j+1}, X(q_{j+1})), \ldots, (\sigma_{m}, X(q_{m}))) \in T(P_{n}^{-1}(pr(K)), G_{1}) \cap T_{m}(G_{1}) \]

\[ \Rightarrow t \in pr(T(P_{n}^{-1}(pr(K)), G_{1}) \cap T_{m}(G_{1})). \] \( \square \)

The following theorem provides the necessary and sufficient condition for the existence of a robust and nonblocking decentralized supervisor as a solution of the problem.

**Theorem 1.** For a nonempty language specification \( K \subseteq \bigcap_{i=1}^{k} P_{n}[L(G_{i})] \), there exists a robust and nonblocking decentralized supervisor \( S_{dec} \) for the uncertain nondeterministic DES \( G := \{G_{1}, \ldots, G_{k}\} \) such that \( P_{n}[L(S_{dec}/G_{j})] = pr(K) \) for all \( G_{j} \in G \) if and only if the following three conditions hold:

(A1) \( K \) is controllable w.r.t. \( \bigcup_{i=1}^{k} P_{n}[L(G_{i})] \).

(A2) \( K \) is coobservable w.r.t. \( \bigcup_{i=1}^{k} P_{n}[L(G_{i})] \) and \( (\Sigma_{c,i}, \Sigma_{c,d,i}, \Sigma_{c,e,i}) \) is I.S.

(A3) \( T(P_{n}^{-1}(pr(K)), G_{i}) = pr(T(P_{n}^{-1}(pr(K)), G_{i}) \cap T_{m}(G_{i})) \) for all \( G_{i} \in G \).

Proof. (If) For \( t \in \Sigma^{*} \), we consider each local supervisor \( S_{i} \) designed as follows: for all \( i \in \{1, \ldots, m\} \),

\[ S_{i}(P_{i}(t)) = \left\{ \begin{array}{l} \sigma \in \Sigma_{c,i} \mid E_{i}(t) \sigma \cup \bigcup_{i=1}^{k} P_{n}[L(G_{i})] \subseteq pr(K) \end{array} \right\} \]

\( \cup \{ \sigma \in \Sigma_{c,i} \mid E_{i}(t) \sigma \cap pr(K) \neq \emptyset \} \cup (\Sigma_{c,e,i} \Sigma_{c,e,i}) \cup \Sigma_{uc} \).

First, we prove that \( P_{n}[L(S_{dec}/G_{j})] = pr(K) \) for any \( G_{j} \in G \). The proof can be done by induction on the length of strings. It holds that \( \varepsilon \in P_{n}[L(S_{dec}/G_{j})] \cap pr(K) \). Let us assume that for any string \( s \) with \( |s| \leq n \), \( s \in P_{n}[L(S_{dec}/G_{j})] \) if only if \( s \in pr(K) \). We first show that \( s\sigma \in pr(K) \).

\[ sa \in P_{n}[L(S_{dec}/G_{j})] \]

\[ \Rightarrow \] There exists \( t \in P_{n}^{-1}(s) \cap L(G_{i}) \) such that \( \sigma \in S_{dec}(t) \)

\[ \Rightarrow \sigma \in P_{n}[S_{dec}[\bigwedge_{i} S_{i}(P_{i}(t)) \cap P_{n}[\bigvee_{i} S_{i}(P_{i}(t))]] \cup \Sigma_{uc}. \]

**Case 1:** \( (\sigma \in \Sigma_{uc}) \) The condition (A1) implies that \( \sigma \in pr(K) \).

**Case 2:** \( (\sigma \in \Sigma_{c,i}) \) We assume that \( s\sigma \notin pr(K) \).

\[ s\sigma \notin pr(K) \Rightarrow \sigma \in P_{n}[L(G_{i})] \backslash pr(K) \]

\[ \Rightarrow \] There exists \( i \in I_{c}(\sigma) \) such that \( E_{i}(s) \sigma \cap pr(K) = \emptyset \)

(by the coobservability of \( K \) w.r.t. \( \bigcup_{i=1}^{k} P_{n}[L(G_{i})] \))

\[ \Rightarrow \sigma \notin S_{i}(P_{i}(t)) \] \( \Rightarrow \sigma \notin P_{n}[\bigwedge_{i} S_{i}(P_{i}(t))] \)

\[ \Rightarrow \sigma \notin S_{dec}(t) \] Contradiction.

**Case 3:** \( (\sigma \in \Sigma_{c,d}) \) \( \sigma \in P_{n}[\bigvee_{i} S_{i}(P_{i}(t))] \)

\[ \Rightarrow \sigma \in \Sigma_{c,d,i} \) and \( \sigma \in S_{i}(P_{i}(t)) \) for some \( S_{i} \)

\[ E_{i}(s) \sigma \cap (\bigcup_{i=1}^{k} P_{n}[L(G_{i})]) \subseteq pr(K) \Rightarrow s\sigma \in pr(K). \]

We next show that \( sa \in pr(K) \) implies \( sa \in P_{n}[L(S_{dec}/G_{j})] \).

\[ \Rightarrow \sigma \in pr(K) \] Let \( sa \in pr(K) \).

**Case 1:** \( (\sigma \in \Sigma_{uc}) \) It follows from the definition of \( S_{dec} \) that \( sa \in P_{n}[L(S_{dec}/G_{j})] \).

**Case 2:** \( (\sigma \in \Sigma_{c,e}) \) For any \( t \in P_{n}^{-1}(s) \) with \( \sigma \in L(G_{i}) \),

\[ E_{i}(t) \sigma \cap pr(K) \neq \emptyset \] for any \( i \in I_{c}(\sigma) \)

\[ \Rightarrow \sigma \in S_{i}(P_{i}(t)) \] \( \Rightarrow \sigma \in P_{n}[\bigwedge_{i} S_{i}(P_{i}(t))] \)

\[ \Rightarrow \sigma \in S_{dec}(t) \] \( \Rightarrow \sigma \in P_{n}[L(S_{dec}/G_{j})] \).

**Case 3:** \( (\sigma \in \Sigma_{c,d}) \) \( \sigma \in pr(K) \)

\[ \Rightarrow \] There exists \( i \in I_{c}(\sigma) \) such that \( E_{i}(s) \sigma \cap L(G_{i}) \neq \emptyset \) for any \( i \in I_{c}(\sigma) \)

\[ \Rightarrow \] \( \sigma \in S_{i}(P_{i}(t)) \) \( \Rightarrow \sigma \in P_{n}[\bigwedge_{i} S_{i}(P_{i}(t))] \)

\[ \Rightarrow \sigma \in S_{dec}(t) \] \( \Rightarrow \sigma \in P_{n}[L(S_{dec}/G_{j})] \).

This completes the proof of \( P_{n}[L(S_{dec}/G_{j})] = pr(K) \) for all \( G_{j} \in G \).
We now prove the robust nonblockingness of $S_{\text{dec}}$. First, $P_n[L(S_{\text{dec}}/G_i)] = pr(K)$ and $A \in S_{\text{dec}}(t)$ for all $t \in \Sigma^*$ imply that $L(S_{\text{dec}}/G_i) = P_{n-1}(pr(K)) \cap L(G_i)$ for all $G_i \in G$. According to Lemma 1, $T(P_{n-1}(pr(K)), G_i) = pr(T(P_{n-1}(pr(K)), G_i) \cap T_m(G_i))$ for all $G_i \in G$.

$\Rightarrow$ For all $G_i \in G$ and for all $s \in P_{n-1}(pr(K))$ and $q \in \delta_i(q_0^i, s)$, there exists $s' \in \Sigma^*$ such that $ss' \in P_{n-1}(pr(K))$ and $\delta_i(q, s') \cap \overline{Q^m} \neq \emptyset$.

$\Rightarrow$ For all $G_i \in G$ and for all $s \in L(S_{\text{dec}}/G_i)$ and $q \in \delta_i(q_0^i, s)$, there exists $s' \in \Sigma^*$ such that $ss' \in L(S_{\text{dec}}/G_i)$ and $\delta_i(q, s') \cap \overline{Q^m} \neq \emptyset$.

$(\text{since } L(S_{\text{dec}}/G_i) = P_{n-1}(pr(K)) \cap L(G_i))$.

$\Rightarrow S_{\text{dec}}$ is robust and nonblocking.

(Only if) We first prove the coobservability of $K$ (A1). For any $s \in pr(K)$ and $s \in \Sigma_{uc}$, $s \in \bigcup_{t=1}^{|pr(K)|} P_n[L(G_i)]$ implies $s \in pr(P_n[L(G_i)])$ for some $G_i$. Since $s \in \Sigma_{uc}$, $s \in P_n[L(S_{\text{dec}}/G_i)]$, and therefore it follows from $P_n[L(S_{\text{dec}}/G_i)] = pr(K)$ that $s \in pr(K)$. We next prove the coobservability of $K$ (A2) by showing a contradiction.

(i) For some $s \in pr(K)$ and $s \in \Sigma_{c,e}$ with $s \in \bigcup_{t=1}^{|pr(K)|} P_n[L(G_i)] \backslash pr(K)$, there is no $i \in I_c(\sigma)$ such that $E_i(s) \cap pr(K) = \emptyset$.

(ii) For some $s \in pr(K)$ and $s \in \Sigma_{c,d}$ with $s \in pr(K)$, there is no $i \in I_c(\sigma)$ such that $E_i(s) \cap pr(K) = \emptyset$.

For any $s \in P_{n-1}(s)$ with $i \in I_c(\sigma)$, we have $E_i(s) \cap pr(K) = \emptyset$.

$\Rightarrow$ For any $i \in I_c(\sigma)$, $E_i(s) \cap pr(K) = \emptyset$.

$\Rightarrow$ For any $s \in P_{n-1}(s)$ with $i \in I_c(\sigma)$, $E_i(s) \cap pr(K) = \emptyset$.

$\Rightarrow$ For some $s \in P_{n-1}(s)$ with $i \in I_c(\sigma)$, $E_i(s) \cap pr(K) = \emptyset$.

We next discuss the computational complexity of verifying the conditions presented in Theorem 1. Let $|Q^G|$ and $|Q^K|$ be the sizes of the state sets of the deterministic automata that recognize $\bigcup_{i=1}^{|Q^G|} P_n[L(G_i)]$ and $K$, respectively. For the condition (A1) in Theorem 1, checking the controllability of $K$ requires the computational complexity of $O(|Q^G| \cdot |Q^K|)$ as we can infer this from the controllability test algorithm of Cassandras and Lafortune (1999). Next, let us discuss the computational complexity of verifying the condition (A2). For simplicity, we consider a case with two local supervisors, but the result can be extended to the case with any finite number of local supervisors. According to the results of Rudie and Willems (1995) and Yoo and Lafortune (2002), $C$ & $P$ and $D$ & $A$ coobservabilities are verifiable in polynomial time by constructing special nondeterministic automata $M_c(\Sigma_c)$ and $M_d(\Sigma_c)$, respectively. Following the approach proposed by Rudie and Willems (1995), it turns out that the algorithm constructing $M_c(\Sigma_c)$ from $\bigcup_{i=1}^{|Q^G|} P_n[L(G_i)]$ and $K$ requires the computational complexity of $O(n)$ where $n = |Q^G| \cdot |Q^K| \cdot |Q^G| \cdot |Q^K|$. Deciding the $C$ & $P$ coobservability of $K$ is equivalent to verifying whether the language recognized by $M_c(\Sigma_c)$ is nonempty. This requires the computational complexity of $O(n^2)$ (Rudie & Willems, 1995). The $D$ & $A$ coobservability can be verified by constructing $M_d(\Sigma_c)$ (Yoo & Lafortune, 2002) and this requires the computational complexity of $O(n^2)$ where $n = |Q^G| \cdot |Q^K| \cdot |Q^G| \cdot |Q^K|$. Finally, let us discuss the computational complexity of verifying the condition (A3). To obtain the trajectory model $T(P_n^{-1}(pr(K)), G_i)$ of any $G_i \in G$, we need to search the state space of $G_i$ for all strings in $pr(K)$ and to compute the related refused events. We note that the internal event $A$ defined at each state of $G_i$ should be then always included in the computation of trajectories. Hence, $T(P_n^{-1}(pr(K)), G_i)$ requires the computational complexity of $O(|Q^K| \cdot |Q^K|)$.

The supervisory existence problem for a given language specification addressed in this paper can also be solved by converting the given nondeterministic finite automata into corresponding deterministic automata. As presented by Heymann and Lin (1997), the following two procedures are required for this. The first step is the extension of a given nondeterministic finite automaton $G_i$ into a deterministic automaton $\tilde{G}_i$ by introducing some hypothetical uncontrollable and unobservable events (‘Extend algorithm’). The second step is the transformation of a given language specification $K$ into a corresponding language $\tilde{K}$ in the lifted deterministic system. The extension of $G_i$ into $\tilde{G}_i$ adds at most $|Q_i| \cdot |\Sigma|$ states, and the lifted automaton $\tilde{G}_i$ has, therefore, at most $|Q_i| \cdot (|\Sigma| + 1)$ states (Heymann & Lin, 1997).
and (A2) in Theorem 1 can be then checked with the conditions (A1) and (A2) in Theorem 1 can be then checked with the sizes of the state sets of the deterministic automata that recognize the extended K also has a state set with the size of |Q/K| = (|Σ| + 1) at most. After these two steps, the controllability, coobservability, and Lm(Ĝi)-closedness of K can be examined over the deterministic automata Ĝi’s. Let |Q̃G| and |Q̃K| be the sizes of the state sets of the deterministic automata that recognize (|Σ| + 1) at most. After these two steps, the controllability, coobservability, and Lm(Ĝi)-closedness of K can be examined over the deterministic automata Ĝi’s. Let |Q̃G| and |Q̃K| be the sizes of the state sets of the deterministic automata that recognize |Q̃G| and |Q̃K|, respectively. The conditions (A1) and (A2) in Theorem 1 are confirmed from the observation that there always exists at least one string reachable to the marked states for G1, G2, from all the reachable states by the strings of Pn−1(pr(K1)).

We consider another specification as follows: K2 = {abc, acc, abc} ⊆ \( \bigcap_{i=1}^{2} P_n[L(G_i)] \). For s = ε ∈ pr(K2) and b ∈ \( \Sigma_{c.e} \), it is true that b ∈ \( \bigcup_{n=1}^{3} P_n[L(G_i)] \) for pr(K2), Lc(b) = {1}, and E1(s)b ∩ pr(K2) = {ε, c}b ∩ pr(K2) = {cb} ≠ Ø. Hence, K2 is also not coobservable with the computational complexity of \( O(n^2) \) and \( (\Sigma_{o,i}, \Sigma_{c.d,i}, \Sigma_{c.e,i}) \in \mathcal{E}_2 \). Moreover, the condition (A3) for G1 in Theorem 1 is not satisfied due to the following reason:

\[
T(P_n^{-1}(pr(K2)), G_1) = pr([\{\emptyset, \{a\}, \{c\}, \{a, \Sigma_n\}], \\
(\emptyset, \{a\}, \{b\}, \{a\})), \{a, \Sigma_n\}), \\
(\emptyset, \{a\}, \{b\}, \{a\})), \{c, \Sigma_n\}), \\
(\emptyset, \{a\}, \{b\}, \{a\})), \{b, \Sigma_n\}), \\
(\emptyset, \{a\}, \{b\}, \{a\})), \{a, \Sigma_n\})).
\]

That is, for the trajectory t = (θ, (c, [a, b], \{b, \Sigma_n\}), (Δ, [b, c])), it holds that t ∈ T(Pn−1(pr(K2)), G1) but t /∈ Tm(G1).

Finally, we consider a specification given by K3 = {abc, acc, bac} ⊆ \( \bigcap_{i=1}^{2} P_n[L(G_i)] \). It can be easily verified that K3 satisfies all of the conditions presented in Theorem 1, and therefore there exists a robust and nonblocking decentralized supervisor Sdec such that Pn[L(Sdec/G1)] = pr(K3) and Pn[L(Sdec/G2)] = pr(K3). In this case, the local supervisors S1 and S2 can be designed as follows:

\[
S_1(P_1(\emptyset)) = S_1(\emptyset) = \{a, A\}, \quad S_2(P_1(\emptyset)) = S_2(\emptyset) = \{a, b, A\}, \\
S_1(P_1(a)) = S_1(a) = \{a, b, A\}, \quad S_2(P_1(a)) = S_2(\emptyset) = \Sigma, \\
S_1(P_1(ab)) = S_1(ab) = \{a, A\}, \quad S_2(P_2(ab)) = S_2(\emptyset) = \Sigma, \\
S_1(P_1(ac)) = S_1(ac) = \{a, b, A\}, \quad S_2(P_2(ac)) = S_2(ac) = \{a, b, A\}, \\
S_1(P_1(abc)) = S_1(abc) = \{a, A\}, \quad S_2(P_2(abc)) = S_2(ac) = \Sigma, \\
S_1(P_1(aa)) = S_1(aa) = \{a, A\}, \quad S_2(P_2(aca)) = S_2(aca) = \Sigma.
\]

5. Conclusions

For an uncertain DES modeled by a set of possible nondeterministic automata, we have shown that the controllability and coobservability of a given language specification can guarantee the existence of a set of local supervisors satisfying the

4. An illustrative example

We consider the uncertain DES G = {G1, G2} shown in Fig. 2 where

\[
\Sigma_n = \{a, b, c\}, \quad \Sigma = \Sigma_n \cup \{a\} = \{a, b, c, A\}, \\
\Sigma_{o,1} = \Sigma_{c,1} = \{a, b\}, \\
\Sigma_{o,2} = \Sigma_{c,2} = \{a, c\}, \quad \Sigma_{c,d} = \{a, b\}, \quad \Sigma_{c,e} = \{b\}.
\]

Since Σn = Σ in this example, any language specifications are controllable w.r.t. \( \bigcup_{i=1}^{2} P_n[L(G_i)] \). First, we assume that a specification is given by K1 = {abc, acc, bac} ⊆ \( \bigcap_{i=1}^{2} P_n[L(G_i)] \). For s = ab ∈ pr(K1) and c ∈ Σc.d.2, it is true that ab ∈ pr(K1), Lc(c) = {2}, and

\[
E_2(s)c \cap \bigcup_{i=1}^{2} P_n[L(G_i)] = \{a, ab, ba\}c \cap \bigcup_{i=1}^{2} P_n[L(G_i)] = \{ab, abc, bac\} \not\subseteq pr(K1).
\]

Hence, K1 is not coobservable w.r.t. \( \bigcup_{i=1}^{2} P_n[L(G_i)] \) and \( (\Sigma_{o,i}, \Sigma_{c,d,i}, \Sigma_{c,e,i}) \in \mathcal{E}_2 \). However, it is evident that the condition (A3) in Theorem 1 is satisfied for K1. This can be confirmed from the observation that there always exists at least one string reachable to the marked states for G1, G2, from all the reachable states by the strings of Pn−1(pr(K1)).
given specification. We have further shown that the trajectory property of a language must be examined to ensure the existence of a robust and nonblocking decentralized supervisor.

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